POSCAT Seminar 8 : Dynamic Programming 2

yougatup @ POSCAT

Topic

- Topic today
  - Dynamic Programming
    - Longest Palindrome
    - Team Division

Longest Palindrome

- Problem

  A palindrome is a word that reads the same forward or reversed

  Given string, find the length of longest palindrome among the subsequence.

b a b a c v a b b a

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1. Define the Table

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1. Define the Table

   let $T(i, j)$ = the length of longest palindrome on $a_i \sim a_j$

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  2. Find a recurrence relation

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2. Find a recurrence relation

Consider the case when $a_i = a_j$ or not

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2. Find a recurrence relation

$$T(i,j) = \begin{cases} T(i+1, j-1) + 2 & if\ a_i = a_j \\ \max(\ T(i+1,j), T(i,j-1)\ ) & otherwise \end{cases}$$

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  3. Calculate !

     Think carefully about the filling direction

Team Division

- Problem

  Suppose that we can divide n numbers into m chunks.

  Minimize the maximum value of sum of each chunk.

  For example, let $m = 3$

  1 2 5 3 4 2 6 7 3 4

Team Division

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  3 20 14

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  ∴ we get 20

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Team Division

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  For example, let $m = 3$

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  ∴ we get 14 → much better !

Team Division

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  1. Define the Table

Team Division

- Problem

  Suppose that we can divide n numbers into m chunks.

  Minimize the maximum value of sum of each chunk.

  1. Define the Table

  $T(i,j) =$ the minimum value when we divide $a_1 \sim a_j$ into i chunks

Team Division

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  2. Find a recurrence relation

Team Division

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  Suppose that we can divide n numbers into m chunks.

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  2. Find a recurrence relation

  $$T(i,j) = \min(\max(T(i-1,k), Cost(k+1,j)))$$

  $$for\ all\ \ i \le k \le j$$

  After making i-1 chunks from $a_1 \sim a_k$, make ONE chunk as $a_{k+1} \sim a_j$ for all possible k.

Team Division

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  3. Calculate !