POSCAT Seminar 7 : Dynamic Programming 1

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Topic

- **Topic today**
  - Dynamic Programming
    - Basic Concept
    - Longest Increasing Subsequence
    - Longest Common Subsequence

Dynamic Programming

- **Problem Solving Paradigm**
  - Identifying a collection of **subproblems**
  - Tackling them one by one
    - smallest first
    - using the answer **already calculated** to help figure out the larger ones
    - until the whole problem is solved

- **One of the most prominent issue**
  - You can be **familiar** with PS with Dynamic Programming
  - There are so many issues to cover

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Dynamic Programming

- **Procedure**

  1. Define the Table

  2. Find recurrence relation

  3. Calculate !

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Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  5   2   8   6   3   6   9   7

Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <div align="center">

  5   **2**   8   6   **3**   **6**   **9**   7

  </div>

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Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <div align="center">5   **2**   8   6   **3**   **6**   **9**   7</div>

  1. Define the Table

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Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <p align="center">5   <b style="color:red">2   8   6   <b style="color:red">3   <b style="color:red">6   <b style="color:red">9   7</p>

  1. Define the Table

  Let $T(i)$ = The length of LIS when a_i is the last element

  $\therefore a_i$ have to be last element of subsequence

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Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <p style="text-align:center">5   **2**   8   6   **3**   **6**   **9**   7</p>

  1. Define the Table

  You can fill the table manually. What is the answer ?

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <p style="text-align:center">5   **2**   8   6   **3**   **6**   **9**   7</p>

  1. Define the Table

     Can you see the recurrence relation on T ?

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- Problem

  Find an increasing subsequence of greatest length

  <p style="text-align:center">5   **2**   8   6   **3**   **6**   **9**   7</p>

  1. Define the Table

     Suppose that $j < i$ and $Data(j) < Data(i)$.

     Then we can add one more element to Increasing Subsequence whose last element is $Data(j)$

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

Longest Increasing Subsequence

- Problem

  Find an increasing subsequence of greatest length

  5   **2**   8   6   **3**   **6**   **9**   7

  1. Define the Table

  Consider element 6 in the example.

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

Longest Increasing Subsequence

- ## Problem

Find an increasing subsequence of greatest length

5   **2**   8   6   **3**   **6**   **9**   7

1. Define the Table

We can make a Increasing Subsequence by adding 6 whose last element is 5. ∴ we can make "5 6"

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  5   **2**   8   6   **3**   **6**   **9**   7

  1. Define the Table

     However, we can make another Increasing Subsequence by considering 2 ! Still the length is 2

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <p style="text-align:center">5   **2**   **8**   6   **3**   **6**   **9**   7</p>

  1. Define the Table

     We can't make IS with 8 because 8 > 6

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <p style="text-align:center">5   <b style="color:red">2   8   <b style="color:blue">6   <b style="color:red">3   <b style="color:red">6   <b style="color:red">9   7</p>

  1. Define the Table

     We can't

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <div align="center">5   **2**   8   6   **3**   **6**   **9**   7</div>

  1. Define the Table

     We can make ! Therefore, our length is 3.

     Note that we considered all the case.

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

Longest Increasing Subsequence

- ## Problem

Find an increasing subsequence of greatest length

<div align="center">5   **2**   8   6   **3**   **6**   **9**   7</div>

2. Find a recurrence relation

$$T(i) = \max(T(j) + 1) \quad for\ all \quad j < i\ \&\ Data(j) < Data(i)$$

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

▪ Problem

Find an increasing subsequence of greatest length

<p align="center">5   <strong style="color:red">2   8   6   <strong style="color:red">3   <strong style="color:red">6   <strong style="color:red">9   7</p>

3. Calculate !

You have to think about where the answer is

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- Problem

  Find an increasing subsequence of greatest length

  5   **2**   8   6   **3**   **6**   **9**   7

  3. Calculate !

  You have to think about where the answer is

  It must be the maximum value of whole Table

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- ## Problem

Find an increasing subsequence of greatest length

<div align="center">

5   **2**   8   6   **3**   **6**   **9**   7

</div>

Well, we can calculate the length of LIS.

Can we find subsequence itself? i.e. "2 3 6 9" in the example.

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  <div align="center">5   **2**   8   6   **3**   **6**   **9**   7</div>

  Well, we can calculate the length of LIS.

  Can we find subsequence itself? i.e. "2 3 6 9" in the example.

  That's possible by writing history !

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- Problem

  Find an increasing subsequence of greatest length

  <div align="center">5   **2**   8   6   **3**   **6**   **9**   7</div>

  What is the answer on this example ?

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- ## Problem

Find an increasing subsequence of greatest length

<p align="center">5   <b style="color:red">2   8   6   <b style="color:red">3   <b style="color:red">6   <b style="color:red">9   7</p>

What is the answer on this example ?

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- Problem

  Find an increasing subsequence of greatest length

$$5 \quad \textbf{2} \quad 8 \quad 6 \quad \textbf{3} \quad \textbf{6} \quad \textbf{9} \quad 7$$

  What is the answer on this example ?

  Then the last element of LIS must be 9 because the definition of $T(i)$ said that a_i should be the last element !

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- Problem

Find an increasing subsequence of greatest length

5   **2**   8   6   **3**   **6**   **9**   7

Well, 9 is definitely last element of LIS.

How about other elements on LIS ?

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- ## Problem

Find an increasing subsequence of greatest length

5   **2**   8   6   **3**   **6**   **9**   7

Well, 9 is definitely last element of LIS.

How about other elements on LIS ?

$T(6)$ is 4 because we add 9 to the end of IS whose last element is 6

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

Longest Increasing Subsequence

- ## Problem

Find an increasing subsequence of greatest length

<p style="text-align:center">5   <strong style="color:red">2   8   6   <strong style="color:red">3   <strong style="color:red">6   <strong style="color:red">9   7</p>

Therefore, 6 have to be located on the left of 9

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence
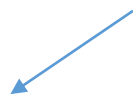
- ## Problem

Find an increasing subsequence of greatest length

5   **2**   8   6   **3**   **6**   **9**   7

Therefore, 6 have to be located on the left of 9

Repeat this procedure until we have no former element

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4

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Longest Increasing Subsequence

- ## Problem

Find an increasing subsequence of greatest length

5   **2**   8   6   **3**   **6**   **9**   7

Therefore, 6 have to be located on the left of 9

Repeat this procedure until we have no former element

It has no former element !

index	0	1	2	3	4	5	6	7
Data	5	**2**	8	6	**3**	**6**	**9**	7
Table	1	**1**	2	2	**2**	**3**	**4**	4

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Longest Increasing Subsequence

- ## Problem

Find an increasing subsequence of greatest length

<div align="center">

5   **2**   8   6   **3**   **6**   **9**   7

</div>

We call this procedure as "**Backtrace**"

To implement Backtrace, we have to write down the history.

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4
Hist.	None	None	1	1	1	4	5	5

It means that former element of a_5 is a_4 !

Longest Increasing Subsequence

- ## Problem

  Find an increasing subsequence of greatest length

  5   **2**   8   6   **3**   **6**   **9**   7

  Implement it ☺. It is very easy to make a stress of yours

index	0	1	2	3	4	5	6	7
Data	5	2	8	6	3	6	9	7
Table	1	1	2	2	2	3	4	4
Hist.	None	None	1	1	1	4	5	5

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

S	N	O	W	Y
S	U	N	N	Y

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

S	**N**	O	W	**Y**
S	U	**N**	N	**Y**

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

  S **N** O W **Y**

  S U **N** N **Y**

  1. Define the Table

     Give me !

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

  S **N** O W **Y**

  S U **N** N **Y**

  1. Define the Table

     Let $T(i, j)$ = The length of LCS between $(a_1 \sim a_i)$ and $(b_1 \sim b_j)$

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

  S **N** O W **Y**
  S U **N** N **Y**

  1. Define the Table   Fill it manually !

	S	N	O	W	Y
S					
U					
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S **N** O W **Y**

S U **N** N **Y**

1. Define the Table   How can we feel it ?

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1			
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y
S U N N Y

Consider this situation.

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1			
N					
N					
Y					

Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

  S N O W Y

  S U N N Y

  First of all, we can't "match" O and U because they are different

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1			
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y

S U N N Y

Therefore, we can remove one or both element

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1			
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y
S U N N Y

What is the length of LCS when we remove O ?

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1			
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y
S U N N Y

It knows !

What is the length of LCS when we remove O ?

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1			
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y

S U N N Y

What is the length of LCS when we remove U ?

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1			
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y

S U N N Y

It has !

What is the length of LCS when we remove U ?

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1			
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y
S U N N Y

Therefore, the value of this entry is 1

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1	1		
N					
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S   N   O   W   Y

S   U   N   N   Y

How about this situation ?

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1	1	1	1
N	1				
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y

S U N N Y

We can match N! because they are the same

	S	**N**	**O**	**W**	**Y**
S	1	1	1	1	1
U	1	1	1	1	1
N	1				
N					
Y					

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Longest Common Subsequence

- ## Problem

Find an Longest Common Subsequence

S N O W Y

S U N N Y

Then, the LCS must be (LCS between "S" and "SU") + N

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1	1	1	1
N	1				
N					
Y					

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

  S N O W Y

  S U N N Y

  Therefore, we get SN. In addition, the value of this entry is 2

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1	1	1	1
N	1	2			
N					
Y					

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

  S N O W Y
  S U N N Y

  Can you see the recurrence relation ?

	S	N	O	W	Y
S	1	1	1	1	1
U	1	1	1	1	1
N	1	2	2	2	2
N	1	2	2	2	2
Y	1	2	2	2	3

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

S	N	O	W	Y
S	U	N	N	Y

  2. Find a recurrence relation

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Longest Common Subsequence

- Problem

  Find an Longest Common Subsequence

  S N O W Y

  S U N N Y

  2. Find a recurrence relation

  $$T(i,j) = T(i-1,j-1) + 1 \qquad\qquad if\ a_i = b_j$$
  $$\max(\ T(i-1,j),\ \ T(i,j-1)\ )\ \ otherwise$$

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Longest Common Subsequence

- ## Problem

  Find an Longest Common Subsequence

  S N O W Y
  S U N N Y

  3. Calculate !

  Think about backtrace. It will be easy if you understand DP well.

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Dynamic Programming

- **Remember. 3 step is enough.**
  - The first step is the most important thing. Finding the recurrence relation is not that hard commonly.
  - I showed somewhat detail explanation of solving procedure because it is first time to you
  - From the next time, I will just let you know what the definition of Table and the recurrent relation is

- **Please solve all the problem**

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