POSCAT Seminar 5 : Recursion and Divide & Conquer

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Topic

- Topic today
  - Recursion
    - Basic Concept
  - Divide & Conquer
    - Basic Concept
    - Finding Medians
    - Closest Pair
    - Implementation

By the way, do you solve all the problems ?

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You should already know what recursion is !

Divide & Conquer

- Problem Solving Paradigm
  1. **Break** the problem into subproblems : smaller instances
  2. Solve them **recursively**
  3. **Combine** the solutions to get the answer to the problem.

- Calculation of time complexity is not trivial
  - We need Master theorem to calculate it
  - Recurrence relation
  - Not that important now

Finding Medians and Selection

- Problem

Find the k-th smallest element in S, where S is a set of numbers

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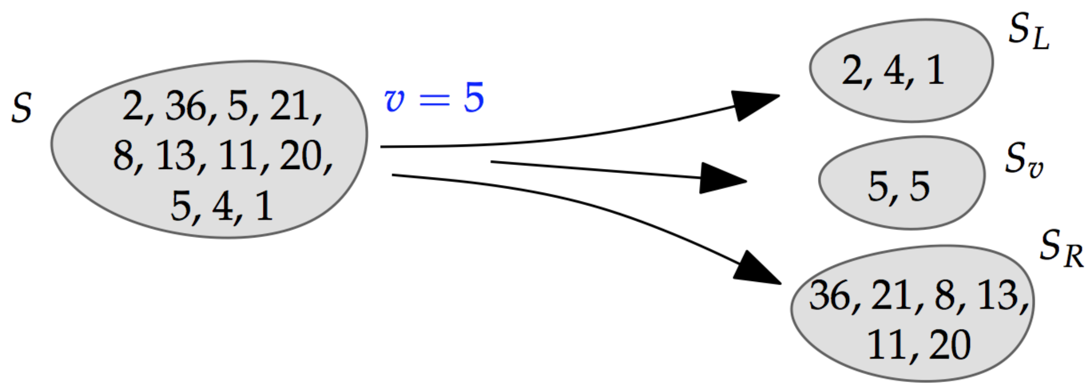
Divide & Conquer : Choose v and split S into S_L, S_v, S_R !



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Divide & Conquer : Choose v and split S into S_L, S_v, S_R !

$$\text{selection}(S,k) = \begin{cases} \text{selection}(S_L,k) & \text{if } k \le |S_L| \\ v & \text{if } |S_L| < k \le |S_L| + |S_v| \\ \text{selection}(S_R, k-(|S_L|+|S_v|)) & \text{if } k > |S_L| + |S_v|. \end{cases}$$

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We have to choose v nicely ! … How ?

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Divide & Conquer : Choose v and split S into S_L, S_v, S_R !

We have to choose v nicely ! … How ? **Randomly !**

$$\text{selection}(S,k) = \begin{cases} \text{selection}(S_L,k) & \text{if } k \le |S_L| \\ v & \text{if } |S_L| < k \le |S_L| + |S_v| \\ \text{selection}(S_R, k-(|S_L|+|S_v|)) & \text{if } k > |S_L| + |S_v|. \end{cases}$$

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If we're lucky, then the partition goes to small very fast. If not, It will take O(n^2). Can we trust random choice of v ?

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Finding Medians and Selection

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If we're lucky, then the partition goes to small very fast. If not, It will take O(n^2). Can we trust random choice of v ?

Yes! because the worst case is extremely unlike to occur !

There are 50% chance of being good, that is, the Sublists S_L and S_R have size at most ¾ of that of S.

Finding Medians and Selection

- Problem

If we're lucky, then the partition goes to small very fast. If not, It will take $O(n^2)$. Can we trust random choice of v ?

Therefore, the expected running time $T(n)$ is

$$T(n) \leq T(3n/4) + O(c \cdot n)$$

where c is the average number of consecutive splits to find a good split. Clearly $c = 2$. So we can conclude that $T(n) = O(n)$

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Closest pair of points

- Problem

Given n points in the plane, find a pair with smallest Euclidean distance between them.

Closest pair of points

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Given n points in the plane, find a pair with smallest Euclidean distance between them.

Brute force approach : Check all pairs of points. O(n^2)

Closest pair of points

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Given n points in the plane, find a pair with smallest Euclidean distance between them.

Divide & Conquer approach

1. Split points into two groups
2. Find closest pair of points for each group
3. Combine the solutions !

Closest pair of points

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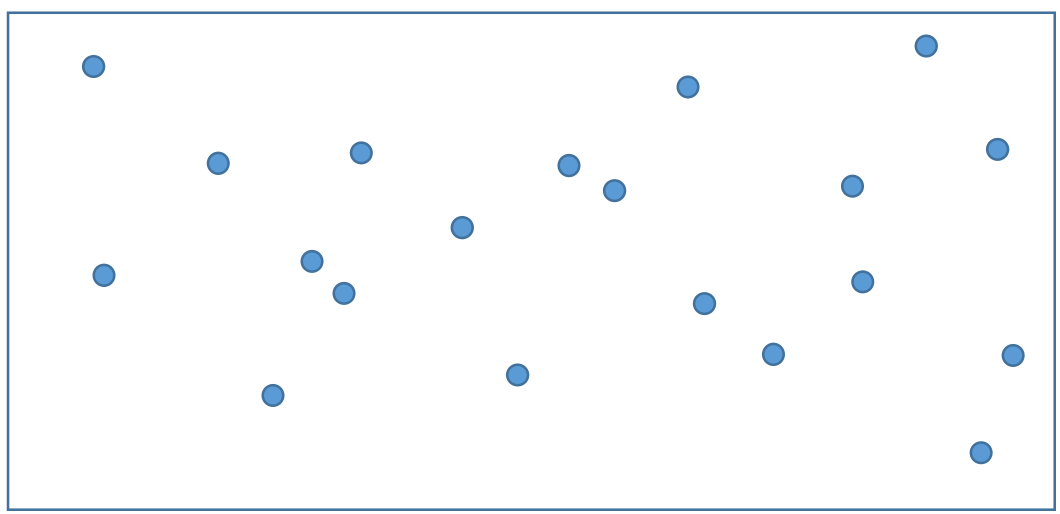

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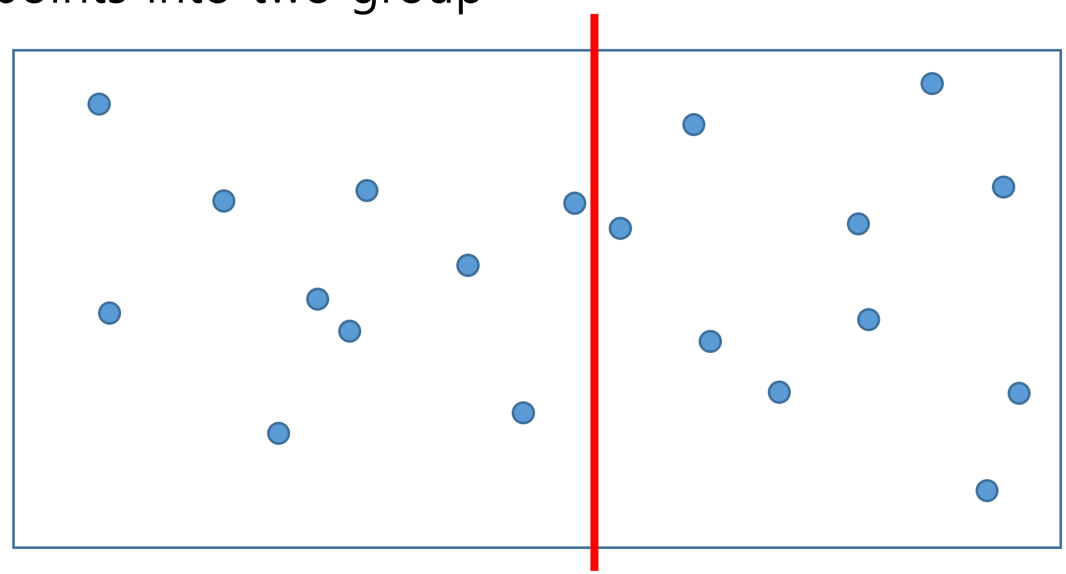

Closest pair of points

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Given n points in the plane, find a pair with smallest Euclidean distance between them.

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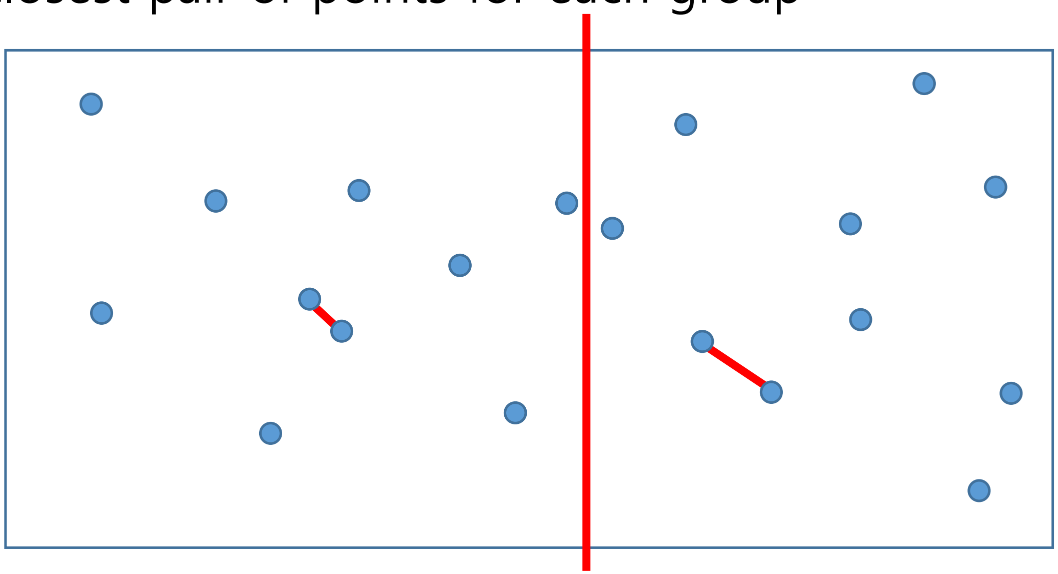

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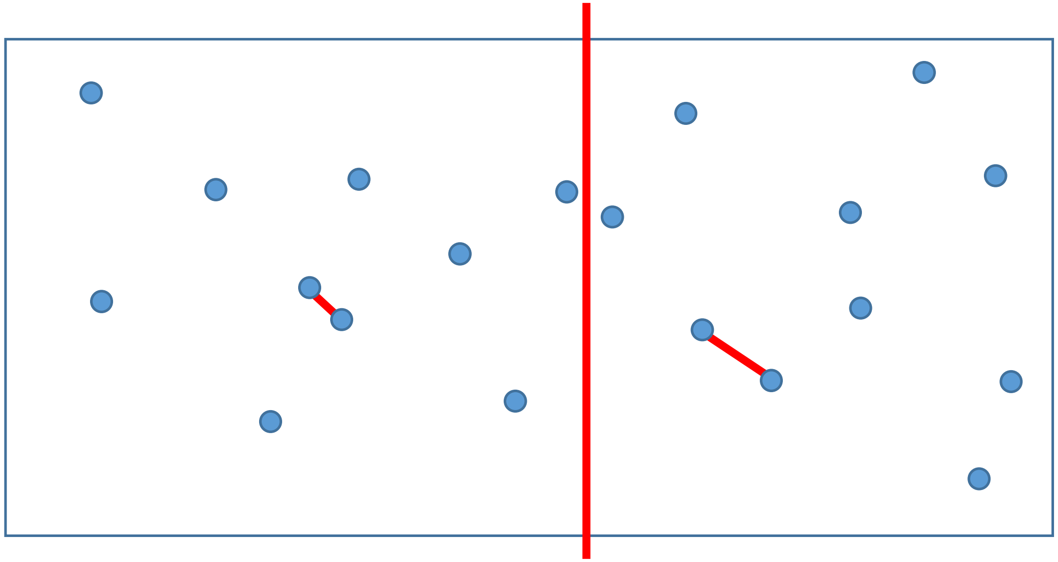
3. Combine the solutions ?



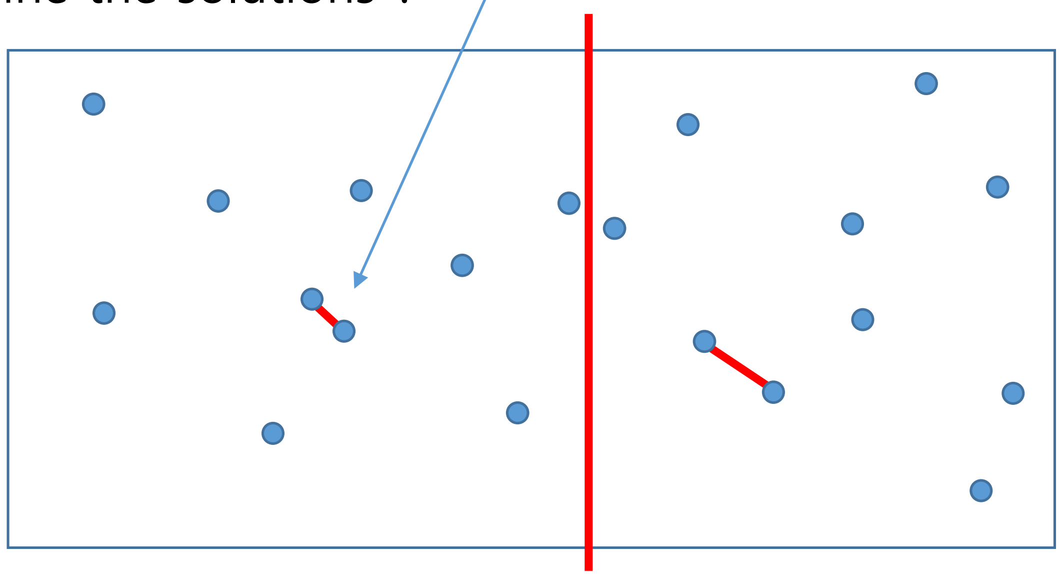
Closest pair of points

- Problem

Given n points in the plane, find a pair with smallest Euclidean distance between them.

Is it enough to choose small one between two solutions ?

3. Combine the solutions ?



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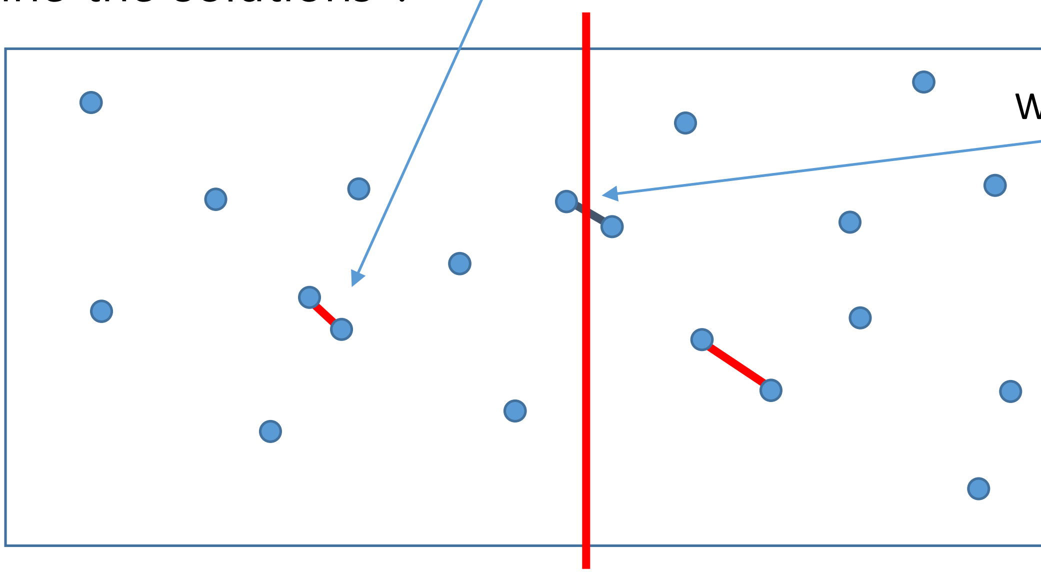
We didn't consider it



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Closest pair of points

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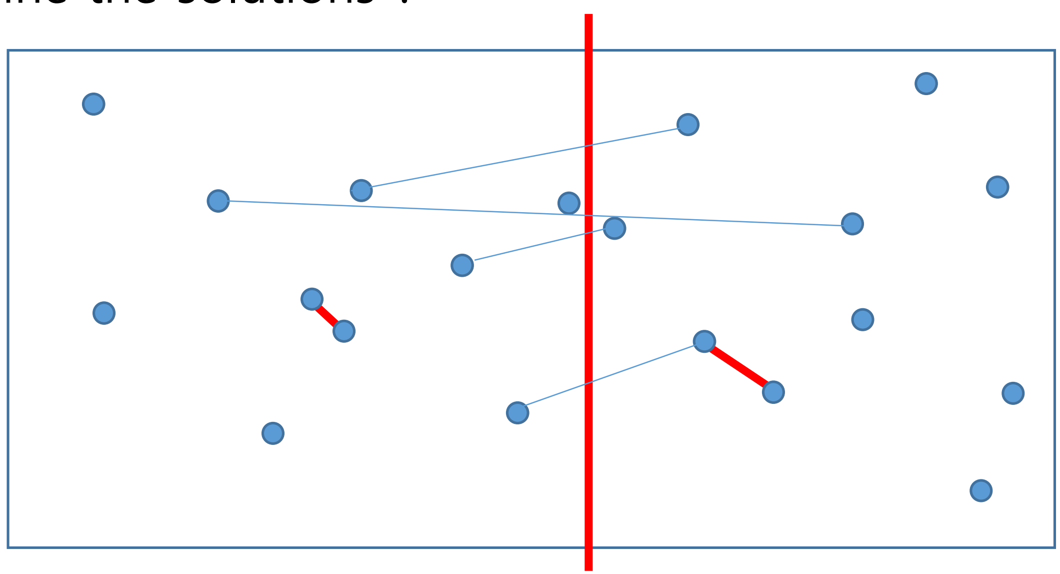
Given n points in the plane, find a pair with smallest Euclidean distance between them.

3. Combine the solutions ?

Then, do we have to consider all the cases when a pair consists of a point in the left and a point in the right ?



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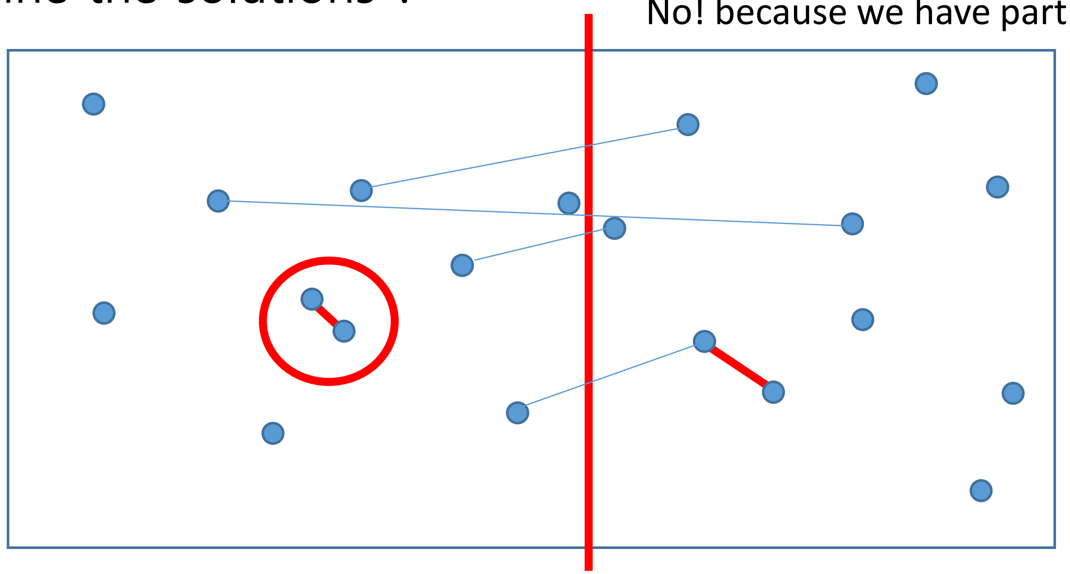
Given n points in the plane, find a pair with smallest Euclidean distance between them.

3. Combine the solutions ?

Then, do we have to consider all the cases when a pair consists of a point in the left and a point in the right ?
No! because we have partial solution !

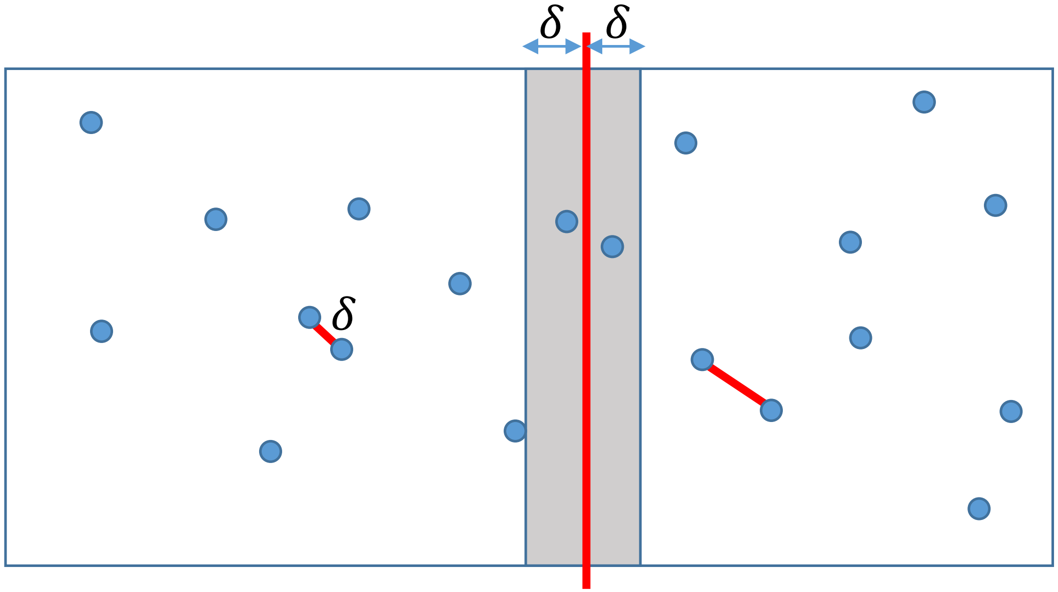


Closest pair of points

- Problem

Given n points in the plane, find a pair with smallest Euclidean distance between them.

Idea : If a closer pair is exist, then it must be in the grey region



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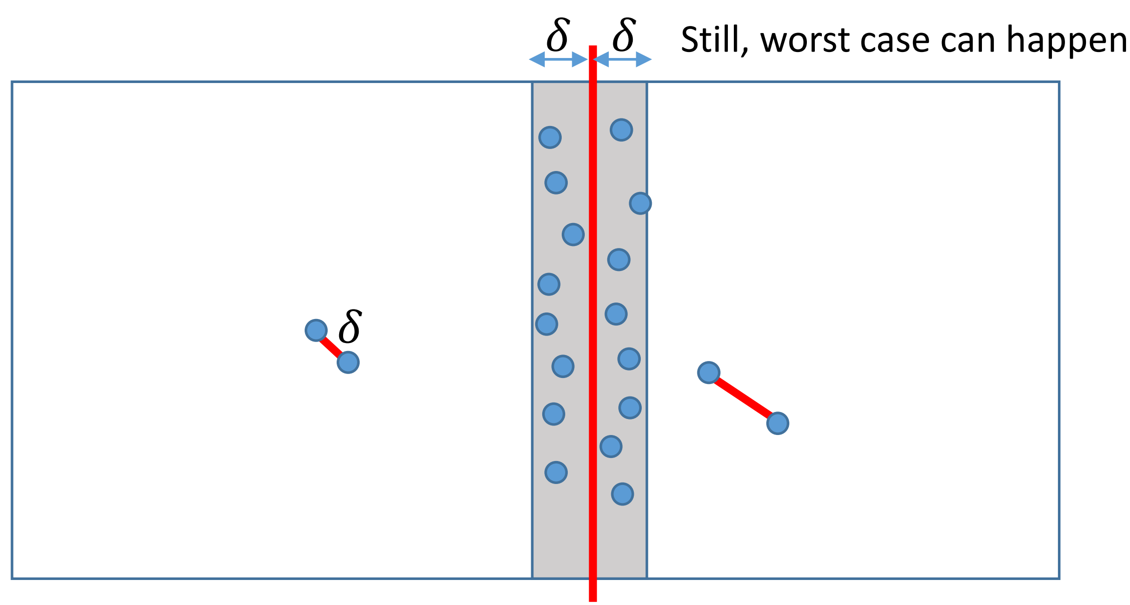


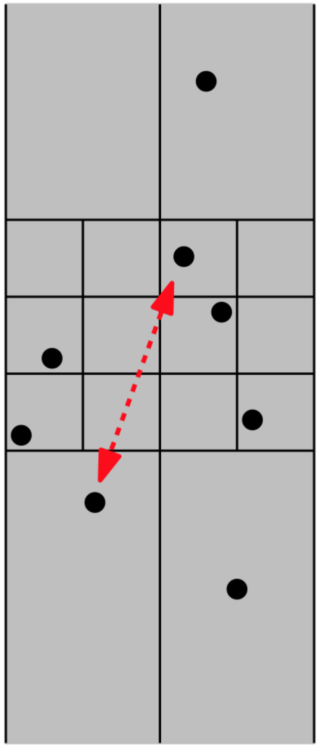
Closest pair of points

- Problem

Given n points in the plane, find a pair with smallest Euclidean distance between them.

Claim. If $|i-j| \geq 12$, then the distance between p_i and p_j is at least δ

- No two points lie in same $\delta/2$-by-$\delta/2$ box.
- Two points at least 2 rows apart have distance $\geq 2(\delta/2)$



Therefore, it is sufficient to consider 12 points above me !

Closest pair of points

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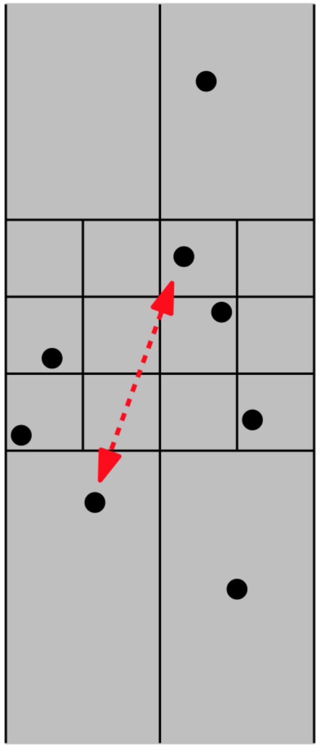
Therefore, it is sufficient to consider 12 points above me ! $\therefore$ T(n) = 2T($n/2$) + O(n) = **O($n \log n$)**



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